Reg. No. : [ ][ ][ ][ ][ ][ ][ ][ ][ ][ ][ ][ ]

**Question Paper Code : 51571**

B.E./B.Tech. DEGREE EXAMINATION, MAY/JUNE 2014.

Third Semester

Civil Engineering

MA 2211/MA 31/MA 1201 A/CK 201/080100008/080210001/10177 MA 301 — TRANSFORMS AND PARTIAL DIFFERENTIAL EQUATIONS/ MATHEMATICS — III

(Common to all branches)

(Regulation 2008/2010)

Time : Three hours Maximum : 100 marks

Answer ALL questions.

PART A — ($10 \times 2 = 20$ marks)

1. State the conditions for a function $f(x)$ to be expanded as a Fourier series in a given interval.

2. Expand $f(x) = 1$ as a half range sine series in the interval $(0, \pi)$.

3. Find the Fourier sine transform of $f(x) = \frac{1}{x}$.

4. State the Fourier integral theorem.

5. Form the PDE by eliminating the arbitrary constants $a, b$ from the relation $z = ax^3 + by^3$.

6. Solve : $(D^4 - D'^4)z = 0$.

7. Write all the solutions of the one-dimensional wave equation $y_{tt} = \alpha^2 y_{xx}$.

8. State the assumptions in deriving the one-dimensions heat flow equation (unsteady state).

9. Find the $Z$-transform of $n^2$.

10. State the convolution theorem on $Z$-transforms.

PART B — (5 × 16 = 80 marks)

11. (a) (i) Expand $f(x) = \begin{cases} 1 + \frac{2x}{\pi}, & -\pi < x < 0 \\ 1 - \frac{2x}{\pi}, & 0 < x < \pi \end{cases}$ as a full range Fourier series in the interval $(-\pi, \pi)$. Hence deduce that $\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \ldots\infty = \frac{\pi^2}{8}$. (8)

(ii) Find the half-range sine series of $f(x) = 4x - x^2$ in the interval $(0, 4)$. Hence deduce the value of the series $\frac{1}{1^3} - \frac{1}{3^3} + \frac{1}{5^3} - \frac{1}{7^3} + \ldots\infty$. (8)

Or

(b) (i) Expand $f(x) = \sin x$ as a complex form Fourier series in $(-\pi, \pi)$. (8)

(ii) Compute the first three harmonics of the Fourier series for $f(x)$ from the following data : (8)

| $x$ : | 0 | $\frac{\pi}{3}$ | $\frac{2\pi}{3}$ | $\pi$ | $\frac{4\pi}{3}$ | $\frac{5\pi}{3}$ | $2\pi$ |
|---|---|---|---|---|---|---|---|
| $f(x)$ : | 1.0 | 1.4 | 1.9 | 1.7 | 1.5 | 1.2 | 1.0 |

12. (a) (i) Find the Fourier transform of $e^{-a|x|}$, $a > 0$ and hence deduce that

(1) $\int_0^\infty \frac{\cos xt}{a^2 + t^2} dt = \frac{\pi}{2a} e^{-a|x|}$

(2) $F\{xe^{-a|x|}\} = i\sqrt{\frac{2}{\pi}} \frac{2as}{(s^2 + a^2)^2}$, here $F$ stands for Fourier transform. (8)

(ii) Solve for $f(x)$ from the integral equation (8)

$$\int_0^\infty f(x) \sin sx\, dx = \begin{cases} 1, & 0 \le s < 1 \\ 2, & 1 \le s < 2 \\ 0, & s \ge 2. \end{cases}$$

Or

(b) (i) Find the Fourier transform of $f(x) = \frac{1}{\sqrt{|x|}}$. (8)

(ii) Using Parseval's identity evaluate the following integrals

(1) $\int_0^\infty \frac{dx}{\left(a^2 + x^2\right)^2}$

(2) $\int_0^\infty \frac{x^2}{\left(a^2 + x^2\right)^2} dx$ where $a > 0$. (8)

13. (a) (i) Form the PDE by eliminating the arbitrary function from the relation $z = y^2 + 2f\left(\frac{1}{x} + \log y\right)$. (8)

(ii) Solve the Lagrange's equation $(x + 2z)p + (2xz - y)q = x^2 + y$. (8)

Or

(b) (i) Solve : $x^2p^2 + y^2q^2 = z^2$. (8)

(ii) Solve : $\left(D^2 + DD' - 6D'^2\right)z = y \cos x$. (8)

14. (a) A string is stretched and fastened to points at a distance '$l$' apart. Motion is started by displacing the string in the form $y = a \sin\left(\frac{\pi x}{l}\right)$, $0 < x < l$, from which it is released at time $t = 0$. Find the displacement at any time $t$. (16)

Or

(b) An infinitely long rectangular plate with insulated surfaces is 10 cm wide. The two long edges and one short edge are kept at 0°C, while the other short edge $x = 0$ is kept at temperature

$u = 20y, \quad 0 \le y \le 5$

$u = 20(10 - y), \quad 5 < y \le 10$.

Find the steady state temperature distribution in the plate. (16)

15. (a) (i) Find the $Z$-transforms of $r^n \cos n\theta$ and $e^{-at} \cos bt$. (8)

(ii) Solve $u_{n+2} - 3u_{n+1} + 2u_n = 4^n$, given that $u_0 = 0$, $u_1 = 1$. (8)

Or

(b) (i) Using convolution theorem find inverse $Z$-transform of $\frac{z^2}{(z-a)(z-b)}$. (8)

(ii) Solve $y_{n+2} - 3y_{n+1} - 10y_n = 0$, given $y_0 = 1$, $y_1 = 0$. (8)

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